Gravitational Trapping for Extended Extra Dimension

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Abstract

The solution of Einsteins equations for 4-brane embedded in 5-dimensional Anti-de-Sitter space-time is found. It is shown that the cosmological constant can provide the existence of ordinary 4-dimensional Newton's low and trapping of a matter on the brane.

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In conventional Kaluza-Klein's picture extra dimensions are curled up to unobservable size. Recently multidimensional models with macroscopic extra dimensions become popular [1]. However, till the present time little attention is paid to multidimensional models with extended extra dimensions, where Universe is considered as a thin membrane in a large-dimensional hyper-Universe [2, 3, 4, 5, 6, 7, 8]. This approach also do not contradict to present time experiments [9].

In the articles [6, 7] we had considered the model of Universe as a 3-bubble expanding in 5-dimensional space-time. Two observed facts of modern cosmology, the isotropic runaway of galaxies and the existence of a preferred frame in Universe where the relict background radiation is isotropic, have the obvious explanation in those models.

Here for the simplicity we again restrict ourselves with the case of five dimensions. The general procedure immediately generalizes to arbitrary dimensionality.

In models with non-compact extra dimensions one needs a mechanism to confine a matter and to explain observed Newton's conventional law inside of 4-dimensional manifold. It is natural to explain this trapping as a result of the special solution of 5-dimensional Einstein's equations [3, 6, 7, 8]. The trapping has to be gravitationally repulsive in nature and can be produced, for example, by large 5-dimensional cosmological constant [3, 6].

As it was shown in our paper [8] the metric tensor, corresponding to the stable splitting of 5-dimensional space-time, slightly generalizes the standard Kaluza-Klein one

$$g_{\alpha\beta} = \lambda^2(x^5)\eta_{\alpha\beta}(x^\nu) \quad , \quad g_{55} = -1 \quad , \quad g_{5\beta} = 0 \quad . \tag{1}$$

Here x^ν are ordinary coordinates of 4-dimensional space-time, $\eta_{\alpha\beta}(x^\nu)$ is 4-dimensional metric tensor and $\lambda^2(x^5)$ is the arbitrary function of fifth coordinate. Solution (1), which in [8] was received from the stability conditions, exactly coincides with the anzats of Rubakov-Shaposhnikov [2].

In this paper we find exact solution of Einstein's equations for 4-brane. This solution is responsible for gravitational trapping of a matter and for the hiding of 4-dimensional cosmological constant resulting effective 4-dimensional Newton's law on the brane.

Simple demonstration of the gravitational trapping and Newton's law restoring mechanism was made by us in the paper [6]. Because of importance of this question we want to repeat here some results of this paper.

As it will be shown below zero-zero component (the only component we need now) of the metric tensor (1) has the form

$$g_{00} = \eta_{00}(x^\mu)e^{E^2|x^5|} \quad . \tag{2}$$

Factor $exp(E^2|x^5|)$ here, which rapidly increases fare from 4-brane $x^5 = 0$, is responsible for the gravitational trapping of a matter.

The integration constant E^2 (corresponding to the width of 4-dimensional world ϵ) in (2) must be taken proportional to 5-dimensional cosmological constant

$$E^2 \sim \Lambda^{1/2} \sim 1/\epsilon \quad , \tag{3}$$

to live 4-dimensional world without the cosmological constant.

Einstein's 5-dimensional equations with the cosmological term for the trapped point-like source in Newton's approximation gives

$$(\Delta - \Lambda)g_{00} = 6\pi^2 GM\delta(r)\delta(x^5) \quad , \tag{4}$$

where Δ is the 4-Laplacian (including derivatives with respect to x^5). Using (2) and (3) and separating variables we obtain ordinary 4-dimensional Newton's formula on the brane without the cosmological term

$$\eta_{00} = 1 - 2gM/r \quad , \tag{5}$$

where

$$g \sim G/\epsilon \tag{6}$$

is 4-dimensional gravitational constant [6].

At the distances of the brane thickness in the right hand side of equation (4) we must put 4-dimensional delta function $\delta(R)$, where R is the radial coordinate in 5-dimensional space-time. So we can't separate the variables and also hide the cosmological constant Λ. The solution of (4) in this case is

$$g_{00} = \frac{2}{z}\left[I_1(z) - \Lambda GM \; K_1(z)\right] \quad , \tag{7}$$

where $z = \Lambda^{1/2}R$ and I_1, K_1 are modified Bessel functions of the order one. In the limit $z \ll 1$ (it means at the distances of the branes width)

$$g_{00} = 1 - 2GM/R^2 \tag{8}$$

and thus 5-dimensional Newton's law is restored [6].

Now we want to consider full system of nonlinear Einstein's equations for the brane with the energy-momentum tensor

$$T_{\mu\nu} = -g_{\mu\nu}\sigma\delta(x^5), \quad T_{55} = T_{\mu 5} = 0 \quad , \tag{9}$$

embedded in 5-dimensional Anti-de-Sitter space-time. Here σ is the tension of the brane.

As it was mentioned above (1) 5-dimensional metric of Universe in Gausian normal coordinates could be written in the form

$$ds^2 = -(dx^5)^2 + \lambda^2(x^5)\eta_{\alpha\beta}dx^\alpha dx^\beta \quad . \tag{10}$$

In these coordinates components of Christoffel's symbol with two or three indices 5 are equal to zero, while with the one index 5 forms the extrinsic curvature tensor [10]

$$\begin{aligned} K_{\alpha\beta} = \Gamma^5_{\alpha\beta} = \frac{1}{2}\partial_5 g_{\alpha\beta} = \lambda\lambda' \eta_{\alpha\beta} \quad , \\ K^{\alpha\beta} = -\frac{1}{2}\partial_5 g^{\alpha\beta} \quad . \end{aligned} \tag{11}$$

Prime denotes derivative with the respect to the coordinate x^5.

Also we would like to represent here some useful relations

$$\begin{aligned} g^{\alpha\gamma}K_{\gamma\beta} = \Gamma^\alpha_{5\beta} = \lambda\lambda' \delta^\alpha_\beta \quad , \\ K = g^{\alpha\beta}K_{\alpha\beta} = g_{\alpha\beta}K^{\alpha\beta} = 4\lambda'/\lambda \quad , \\ \partial_5 K = g^{\alpha\beta}\partial_5 K_{\alpha\beta} - 2K^{\alpha\beta}K_{\alpha\beta} \quad . \end{aligned} \tag{12}$$

Any vector or tensor is naturally split-up into its components orthogonal and tangential to the brane. Using decomposition of the curvature tensor

$$\begin{aligned} {}^5R_{\alpha\beta} &= R_{\alpha\beta} + \partial_5 K_{\alpha\beta} - 2K^{\gamma}_{\alpha}K_{\gamma\beta} + KK_{\alpha\beta} \quad , \\ {}^5R_{55} &= -\partial_5 K - K^{\alpha\beta}K_{\alpha\beta} \quad , \\ {}^5R &= R + K^{\alpha\beta}K_{\alpha\beta} + K^2 + 2\partial_5 K \end{aligned} \tag{13}$$

one can find decomposition of Einstein's equations

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}R - 3\eta_{\alpha\beta}(\lambda\lambda" + \lambda'^2) &= -\eta_{\alpha\beta}\lambda^2[\Lambda + 6\pi^2 G\sigma\delta(x^5)] \quad , \\ R + 12\lambda'^2 &= 2\lambda^2\Lambda \quad . \end{aligned} \tag{14}$$

In four dimensions to have Einstein's equations without the cosmological term

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}R &= 0 \quad , \\ R &= 0 \quad , \end{aligned} \tag{15}$$

we must put

$$\begin{aligned} 3(\lambda\lambda" + \lambda'^2) &= \lambda^2[\Lambda + 6\pi^2 G\sigma\delta(x^5)] \quad , \\ 6\lambda'^2 &= \lambda^2\Lambda \quad . \end{aligned} \tag{16}$$

Using formula

$$|x^5|' = H(x^5) - H(-x^5) \quad , \tag{17}$$

where $H(x^5)$ is the step function, one can show that system (16) has the trapping solution

$$\lambda = e^{E^2|x^5|} \quad , \tag{18}$$

where the integration constant has the value

$$E^2 = \sqrt{\Lambda/6} = \pi^2 G\sigma \quad . \tag{19}$$

This formula also contains necessary relation between the brane tension σ and 5-dimensional cosmological constant Λ.

So we received the solution we have used in (2) to demonstrate our mechanism of gravitational trapping and Newton's law restoration on the brane.

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